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# Aristotle's Formal Language

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## Introduction

Over the last half-century and more, attempts have been made to disengage Aristotle's own logic from the logic that dominated the schools from antiquity forward—the so-called traditional logic. Most of the scholarly attention has been focussed on the system of inferences.<sup>1</sup> It remains to be shown how the propositions furnishing the premisses and conclusions of the inferences are to be construed.

Aristotle's analysis of propositions in the *Prior Analytics* differs from his analysis in the *Categories* and *On Interpretation*. There Aristotle analyzes propositions into verbs and nouns, and this approach suggests a modern function-argument analysis. A distinctive feature of the *Prior Analytics* analysis, on the other hand, is Aristotle's use of ὑπάρχειν as a second-order expression to convey the relation that the terms—not the designata of the terms—of a syllogism have to one another. In the *Prior Analytics*, Aristotle's treatment of propositional structure, in which ὑπάρχειν is not a vague expression but one with a deliberately minimal meaning, is framed to accommodate to syllogistic the propositional structure of the rest of the *Organon*, especially both types of predication distinguished in the *Categories*, *On Interpretation*, and *Topics*—descriptive and definitory.

In the *Prior Analytics* treatment of propositional structure, Aristotle employs a notation consisting at least of his basic three-term schemata of Greek capital placeholder letters<sup>2</sup> but able to be supplemented. For the three authentic figures, these schemata are ΑΒΓ, ΜΝΞ, and ΠΡΣ. These schemata are a limiting case of schema, consisting entirely of blanks, in a spatially-ordered arrangement. It is likely that Aristotle himself used connecting lines in his diagrams to exhibit the relations of the terms to one another, perhaps elaborating these into lune-and-triangles proof forms of the three figures, such as appear from Ammonius forward. I urge that, in the notation that Aristotle used in diagrams, the connecting lines occur where he might have used inflections of ὑπάρχειν in speech or writing. This examination of Aristotle's formal language corrects the view that his logic uses propositions of the subject-copula-predicate form and that the intended interpretation of his syllogistic is a logic of class inclusion and exclusion.

## Logical Syntax in the *Categories* and *On Interpretation*

In the *Categories* and *On Interpretation*, Aristotle's logical syntax includes nouns (names), verbs (predicates), formulae composed of nouns and verbs, truth-functional

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<sup>1</sup> E.g. John Corcoran, "Aristotle's Natural Deduction System," in *Ancient Logic and its Modern Interpretations*, ed. John Corcoran (Dordrecht: Reidel), 1974.

<sup>2</sup> Lynn E. Rose, *Aristotle's Syllogistic* (Springfield, IL: Charles C. Thomas), 1968.

operators, quantifiers, and modal operators.<sup>3</sup> A noun or name (ὄνομα) is for Aristotle “a sound significant by convention, which has no reference to time, and of which no part is significant apart from the rest” (*De Int.* 16a19-21). Some natural sounds, as those made by beasts, are significant; but none of these is a name—a conventional sound (σύμβολον, a28). Names serve as arguments to proposition-forming functors; some inflections of nouns are excluded as not meeting this condition (*De Int.* 16a35-b5). A name need only have sense; Aristotle does not require that it have reference as well. A name may fail to refer either because it names something fictitious or impossible (for example, τραγέλαφος, goat-stag, at *De Int.* 16a17, Aristotle’s ‘Pegasus’), or because it names something which might exist but does not (for example, μὴ ὄντος Σωκράτους, the non-existent Socrates, at *Cat.* 13b17, Aristotle’s ‘the present king of France’).

For Aristotle, a verb or predicate (ῥῆμα) is

that which, in addition to its proper meaning, carries with it the notion of time . . . it is a sign of something said of something else . . . i.e. of something either predicable of or present in some other thing. (*De Int.* 16b6-11).

A verb proper indicates present time; a predicate expression referring to time other than the present is not a verb simply but a tense (πτῶσις) of a verb (*De Int.* 16a16-19).

Verbs and tenses of verbs are proposition-forming functors. No expression, no matter how complex, is a proposition unless it contains a verb (*De Int.* 17a11-15).

A noun or verb standing alone is well-formed to the extent that it is significant. A noun or verb standing alone, however, is not a sentence and has no truth-value (*De Int.* 16a10-19). Verbs not conjoined with arguments are names (ὀνόματα, 16b20). They may stand for states of affairs (πράγματα, cf. τοῦ πράγματος, 16b23), but they make no assertions about states of affairs unless conjoined with arguments.

A sentence (λόγος) is for Aristotle a significant utterance (φωνή σημαντική) whose parts have independent meaning, but only the sentence as a whole makes any affirmation or denial (*De Int.* 16b26-27). Every sentence is significant and signifies by convention. Not every sentence, however, is a declarative sentence (17a3). A prayer, for instance, is a sentence, but not a declarative sentence, since it has no truth-value (17a5-6). Declarative sentences are the subject matter of logic; the study of non-declarative sentences belongs to rhetoric or poetry (17a6-7).

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<sup>3</sup> Peirce picked up on this and remarks that Aristotle “in his treatise upon forms of propositions, the *De Interpretatione*, analyzes the categorical proposition into the *noun*, or *nominative*, and the *verb*.” C. S. Peirce, *Collected Papers*, ed. Charles Hartshorne and Paul Weiss (Cambridge: Harvard University Press), 4.41 Peirce used ‘rhema’ (transliterating Aristotle’s ῥῆμα) as his own technical term for ‘predicate’ in what cannot but be an acknowledgement of Aristotle’s use in *On Interpretation*. *Collected Papers* 2.95. Peirce’s editors note at this point: “Today the rhema, or rheme, is conventionally symbolized as φx and is called a propositional function.”

A sentence is well-formed for Aristotle—a combined expression (λεγόμενον κατὰ συμπλοκήν, cf. *Cat.* 1a16-19)—if it contains a name in the subject place and a verb in the predicate place. A sentence with the name of an individual in the predicate place is ill-formed, as is any sentence with a predicate of lower order than its subject. Only combined expressions are true or false (*Cat.* 2a4-10; *De Int.* 17a3-4).

Aristotle recognizes several varieties of negation. According to him, a negation may be applied to a declarative sentence simply or to a declarative sentence preceded by a modal operator; in either case, negation's role is truth-functional. Negations may be applied also to nouns and verbs. A negation applied to a name yields what Aristotle calls an indefinite name (ὄνομα ἄοριστον, *De Int.* 16a32). A negation applied to a verb yields what Aristotle calls an indefinite verb (ἄοριστον ῥῆμα, 16b15). An indefinite verb may be used both of arguments that refer and of arguments that fail to refer.

It is interesting to note that ὑπάρχειν occurs in both the *Categories* and *On Interpretation* in contexts that support its metalinguistic interpretation. In the *Categories*, where Ackrill was unsatisfied with “the blanket term ‘belong to,’” ὑπάρχειν occurs after 12b29, as he noted, whereas Aristotle has used ‘is in’ or ‘is said of’ before. Here, ὑπάρχειν figures in a discussion of the distinction of positives and privatives from contraries, where Aristotle is clearly concerned with the proper use of opposed expressions and is deriving laws for the assignment of contraries. In *On Interpretation*, ὑπάρχειν is employed at 16a34 in the discussion of “indefinite nouns” (Aristotle's example is ‘not-man’), which, he says, we assign to all sorts of things, both existent and non-existent (ἐφ’ ὅτου οὖν ὑπάρχει καὶ ὄντος καὶ μὴ ὄντος). He uses it again at 16b12-15 in his discussion of “indefinite verbs,” in the identical locution. The message here is that while being, τὸ ὄν, famously for Aristotle, λέγεται πολλαχῶς “is predicated in many senses” (*Metaph.* 1028a10; cf. 1003b6-10, 1017a23-30), τὸ μὴ ὄν, the non-existent, is neither a substance nor an accident, so that it cannot figure as either subject or predicate in Aristotle's theory of definitory and descriptive predications. Yet we can form sentences in which these nameless names of non-existents occur, and for these sorts of sentences which ascribe something to some non-thing or some non-thing to something, Aristotle uses ὑπάρχειν.

Aristotle recognizes quantifying conventions for subjects and for propositions but not for predicates (*De Int.* 17a39-b16). Designata (τῶν πραγμάτων, 17a39) of subject expressions are universal if they are such that their signs can be predicated of many subjects (‘man,’ for instance, designates a universal); they are individual if they are such that their signs cannot be predicated of many subjects (‘Callias,’ for instance, designates an individual).

### **Premises in the *Prior Analytics***

In developing the logical syntax of his syllogistic, Aristotle does not begin with terms and go on to build up propositions with terms and connectors. He starts by defining ‘premiss’ (*An. Pr.* 24a16):

Πρότασις μὲν οὖν ἐστὶ λόγος καταφατικὸς ἢ ἀποφατικὸς τινὸς κατὰ  
τινος· οὗτος δὲ ἢ καθόλου ἢ ἐν μέρει ἢ ἀδιόριστος.

A premiss is an affirmative or negative statement of something about some subject. This statement may be universal or particular or indefinite. (trans. Tredennick)

Aristotle then engages in a discussion of the distinction of syllogistic premisses from demonstrative and dialectical premisses, and only after that, at 24b16, states that “terms” (ὅροι) are that into which premisses can be analyzed—κατηγορούμενον, ‘predicate,’ and καθ’ οὗ κατηγορεῖται, ‘subject.’ No reference is made to a connector. The infinitive ὑπάρχειν appears in Aristotle’s discussion of premisses at 24a18:

λέγω δὲ καθόλου μὲν τὸ παντὶ ἢ μηδενὶ ὑπάρχειν

By universal I mean applies to all, or to none (trans. Tredennick, modified)

Although ὑπάρχειν is widely used in ancient Greek, Aristotle’s constructions with ὑπάρχειν not only strike English speakers as odd, but, as Patzig points out, their unnaturalness in Greek is remarked by Alexander.<sup>4</sup> Patzig and Smith both point out that ὑπάρχειν is artificial, in the sense of being stilted or possibly technical.<sup>5</sup> Patzig considers the reasons that Alexander proposes for Aristotle’s choosing such artificiality. The first is that “because in this way the union of the terms (συναγωγὴ τῶν λόγων) is clear.” Patzig says that he does not understand this but refers in his note 23 to the occurrence in a logical sense in Aristotle of συναγωγὴ only in the *Rhetoric* (1400b26 and 1410a22), where Patzig says that “it means much the same as the logical proof of a proposition.”<sup>6</sup> It clearly means the same in the quotation from Alexander, provided one doesn’t insist on rendering τῶν λόγων “of the terms” as if it were τῶν ὁρῶν, but rather ‘of the sentences’ or ‘of the statements.’

Not only does Aristotle use λόγος consistently for ‘sentence’ in the *Categories* and *On Interpretation*, but, as Prior noticed, the Athenian Stranger in Plato’s *Sophist*, 262C6-7, says that, when you combine a verb with a noun, there is a λόγος that is τῶν λόγων ὁ πρῶτος τε καὶ σμικρότατος—the first and littlest kind of λόγος.<sup>7</sup> This statement of the Stranger’s indicates both that the structure of the simplest sentence

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<sup>4</sup> In *An. Pr. I* in *Commentaria in Aristotelem Graeca*, ed. Maximilian Wallies (Berlin: G. Reimer), 1905, p. 54, 21-29.

<sup>5</sup> Gunther Patzig, *Aristotle’s Theory of the Syllogism*, trans. Jonathan Barnes (Dordrecht: Reidel), 1968, pp. 9-11. Robin Smith in Aristotle, *Prior Analytics*, translated, with introduction, notes, and commentary by Robin Smith (Indianapolis: Hackett Publishing Company), 1989, xviii, “quite artificial;” cf. 107, “The locutions ‘belongs to some,’ etc. are peculiar to Aristotle and not at all everyday Greek.”

<sup>6</sup> Patzig seems to be correct in this. The text at 1400b26 identifies συναγωγὴ with elenctic enthymemes and the text at 1410a22 defines elenchus (indirect proof) as a bringing together (συναγωγὴ) of contraries.

<sup>7</sup> A. N. Prior, *Formal Logic*, 2<sup>nd</sup> ed. (Oxford: Oxford University), 1962, pp. 72-73.

parallels that enunciated by Aristotle and that there was current an Academic definition of a λόγος as consisting of at least two terms.<sup>8</sup> Figuring that this definition was known to Aristotle and was passed down to the Peripatetics, it seems likely Alexander may be understood to be saying that Aristotle's choice of construction makes the proof clearer. Patzig accepts Alexander's second reason, that ὑπάρχειν takes the dative (as "to all" [παντί] and "to none" [μηδενί] above make clear) and thus makes obvious which term is subject and which is predicate. Patzig urges a comparison with the function/argument symbolism of modern mathematical logic, ascribing to Aristotle the same concern to "reveal with all possible clarity the *logical structure* of the propositions which enter the syllogism as premisses or conclusion."<sup>9</sup>

I wish to suggest that ὑπάρχειν is a metalinguistic operator indicating the relation of terms—not objects or the designata of terms—to one another in the premisses of his syllogistic.<sup>10</sup> The *Prior Analytics* premiss construction, if taken to be in the object language, is inconsistent with Aristotle's remarks on predication in the *Categories* and *On Interpretation*, except for a few occurrences of ὑπάρχειν in those works, where the role of the expression is clearly metatheoretical. Aristotle in the *Categories* worked out what are traditionally known as the predicaments, a sorting of names into names of primary and secondary substances and into names falling into the nine categories of accidents, that is to say, names that designate beings of one sort or another. Aristotle has, moreover, in the *Topics*, worked out a scheme of predicables—genus, accident, definition, and property—that, with *differentia*, provide him with tools for analyzing the logical behaviour of premisses that, in the case of the *Topics*, include dialectical premisses. The predicables clearly belong to the language of the metatheory of predication, as they are headings for sorting kinds of predicate expressions considered as actually playing roles in predicative formulae. It is especially obvious in the cases of definition and property, which for Aristotle are products—not conditions—of analysis, that propositions predicating either of these of a subject make no assertion that the definition or property belongs to the subject in the sense that the subject possesses it as it would possess something real.

Aristotle's choice of vocabulary serves to separate the *Prior Analytics* analysis of propositions from that in the rest of the *Organon*, which distinguishes definitory from descriptive predication. The examples that Aristotle supplies as values for his term variables include both definitory predicates like 'animal' and descriptive predicates like

<sup>8</sup> These matters are discussed in J.J. Mulhern's unpublished paper

"ΤΩΝ ΛΟΓΩΝ Ο ΠΡΩΤΟΣ ΤΕ ΚΑΙ ΣΜΙΚΡΟΤΑΤΟΣ (*Sph.* 262C6-7)," 2001.

<sup>9</sup> Patzig 1968, p. 11.

<sup>10</sup> In this view of some of the language of the *Prior Analytics*, I am anticipated by Prior and Rose. As J. J. Mulhern in "Modern Notations and Ancient Logic" in Corcoran (1974) remarks, "Rose followed an aside of Prior (1962) in suggesting that Aristotle had formulated his assertoric analytical syllogisms as inference schemata in the metalanguage rather than as laws in the object language." As Rose himself remarks, "What I am suggesting is, in more modern terms, that many of Aristotle's remarks are in something like a syntactical meta-language: they are not statements of syllogisms, but rather they are statements about syllogisms," p. 25.

‘white,’ which suggests that ὑπάρχειν is chosen as neutral between these two kinds of predication and thus accommodates both of them. It is thus not a vague, untechnical “blanket term”<sup>11</sup> but a technical term, used with a deliberately minimal meaning. It figures, moreover, not only in his assertoric syllogistic but in his modal syllogistic as well.

Further, Aristotle used a sign—made a gesture or drew a line—to represent the relation of terms appearing in a syllogistic figure when he presented his deductive schemata in formal notation when teaching. This is indicated clearly in later diagrams, where inflections of ὑπάρχειν, either as such or modified by modal expressions, are written along the connecting lines.

If, as is urged here, ‘belongs to’ is not, for Aristotle, what we would look at as a dyadic predicate in the object language, then one of the points at which the Peripatetics and the Stoics are thought to have been at loggerheads is resolved. Jonathan Barnes and Michael Frede have written as if ‘belongs to’ were a dyadic predicate in the object language, perhaps misled by ‘belongs to’ as a translation of ὑπάρχειν.<sup>12</sup> Their views are discussed by Phillip Corkum<sup>13</sup> in examining two Stoic examples of unmethodically conclusive arguments, one employing the relation ‘is greater than’ and the other the relation ‘is equal to’ and both depending on the assumption of the transitive law to be transformable into satisfactory deductions, at least for the Stoics. Alexander reports<sup>14</sup> that the Stoics claimed that Aristotelian syllogisms were unmethodically conclusive as well and thus not deductions either. Alexander does not report their reasoning, but Corkum indicates that Frede and Barnes have taken a stab at it, conjecturing that a syllogism in Barbara, for example, employs the relation ‘belongs to every’ and depends for its validity on assuming the *dictum de omni*. Frede goes on to say:

It would be up to the Peripatetics to show that assumptions about ‘belonging to’ are in a logically privileged position whereas assumptions about ‘being equal to’ or ‘being bigger than’ or ‘being a relative of’ are not in that position. But it is difficult to think of any satisfactory argument which would have shown that ‘belonging to’ is in a privileged position and at the same time would not have indicated that other expressions are in the same privileged position and which therefore would have forced the Peripatetics to admit arguments as syllogisms which they did not want to count as such.<sup>15</sup>

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<sup>11</sup> J. L. Ackrill, *Aristotle's Categories and De Interpretatione* (Oxford: Clarendon Aristotle Series), 1963, p. 109, note ad *Cat.* 11b38.

<sup>12</sup> Jonathan Barnes, “Logical Form and Logical Matter,” in *Logica, mente, e persona*, ed. A. Alberti (Florence: Olschki), 1990; Michael Frede, „Stoic vs. Aristotelian Syllogistic,” *Archiv für Geschichte der Philosophie*, 56: 1-32, 1974.

<sup>13</sup> Phillip Corkum, “Is the Syllogistic a Logic?”, International Conference on Ancient and Medieval Logic, Fordham University Lincoln Center, 2003.

<sup>14</sup> *In An.Pr.* 345.15-18.

<sup>15</sup> Frede 1974

As has been argued above, however, ‘belongs to,’ as part of Aristotle’s metalanguage, is in a logically different—if not, perhaps, privileged-- position vis-à-vis ‘is greater than’ and ‘is equal to,’ which are dyadic predicates in the object language. The Stoics are often portrayed as having the advantage of Aristotle in their explicit handling of these dyadic predicates in their logic. It is the case, nevertheless, that these dyadic predicates designate relations, which were known to or perhaps discovered by Aristotle, who placed them in the πρὸς τι slot among the nine accidents in his scheme of categories. Aristotle recognized the category of relation, distinguishing it especially from the category of quantity, and he placed such restrictions on the use of relative premisses in the *Categories* vii as would make them legitimate tools of argument and not just the vehicles of sharp practice that they were, for instance, for Euthydemus and Dionysodorus in Plato’s *Euthydemus*. C. S. Peirce, who knew something about the logic of relations, claimed an Aristotelian precedent for it.<sup>16</sup>

Aristotle, nonetheless, although he recognized and knew how to use premisses with dyadic predicates designating relations among objects, did not have a primary interest in these premisses in his theoretical work. Although accidental predicates, including relative ones, do figure among the examples and discussions in the *Prior Analytics*, Aristotle excluded them from his demonstrative syllogistic. Demonstration for Aristotle requires premisses with their predicates assigned universally (καθόλου), which means that they are assigned both to all (κατὰ παντός) and *per se* (καθ’ αὐτό).<sup>17</sup> These strictures exclude accidental predicates, including relative ones.

### Formal Language in the Graphical Representation of Aristotle’s Schemata

This section builds on Lynn Rose’s 1968 work on graphical representations of Aristotle’s syllogisms as they might have been used in his own lecturing,<sup>18</sup> examining them in commentators and in the ancient classroom. The discussion shows that

- related drawings and diagrams are extant from Aristotle’s time
- his classroom very likely provided for them
- the *Prior Analytics* indicates elements of diagrams known later that could easily have been developed into these diagrams
- diagrams could easily have been lost from the Aristotelian manuscripts and then recovered

The diagrams are shown to have expository, mnemonic, and probative roles, and it is suggested, *contra* Bochenski, that the lune and triangle diagrams for syllogistic proofs and the *pons asinorum* diagram, for the proof of what for Aristotle is akin to a decision procedure, are features integral to his system, making it more likely that he discovered and used them himself.

<sup>16</sup> CP 2.532, 2.577, and 3.643.

<sup>17</sup> *An. Post.* 74a4-5; cf. 73a21-74a3; see M. M. Mulhern, “Aristotle on Universality and Necessity,” *Logique et Analyse*, 12 (1969), pp. 288-299

<sup>18</sup> Rose, Chapter III, Appendices IV and V.



**The Three-term Array.** The *Prior Analytics* presents the first figure of the syllogism as a linear array of Greek capital letters, placeholders for the terms. Rose has established that Aristotle used a three-term array of letters to represent just the premisses of a syllogism, with the conclusion shown below as a separate line.<sup>19</sup> For Aristotle's three genuine figures, this gives, using the Greek capitals in the text of the *Prior Analytics* for each:

(1)	(2)	(3)
A B Γ	M N Ζ	Π Ρ Σ
A Γ	N Ζ	Π Ρ

Rose points out that these concise three-letter forms are offered by Aristotle at 42b21, τῷ ABΓ ('by the [syllogism] ABΓ'), and at 65b4, τῶν ABΓ ('of the [terms] ABΓ'), where the inflection of the Greek definite article shows that the first is singular while the second is plural, so that Aristotle could use this notation either to refer to a single syllogism or to refer to its three terms. Rose remarks:

When Aristotle has occasion to discuss a syllogism, or to explain its validity or invalidity, he nearly always has to present more information about such matters as quantity, quality, figure, etc., than could be expressed by the three letters alone. . . . The three-letter notation ABΓ still represents the basic structure of the syllogism; but it is so concise that it rarely tells us all we need to know about a syllogism, and I suggest that this is why Aristotle has so little occasion to use it in his writings.<sup>20</sup>

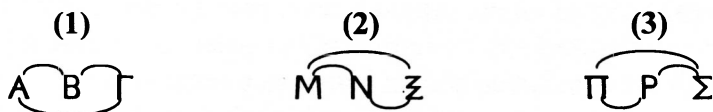
The same evidence shows why Aristotle would have had so much occasion to use his concise notation to keep the basic matter before the students' eyes while he made his points about them—the same as in a logic classroom today. As Rose suggests, however, more information has to be supplied, and there are indications in the *Prior Analytics* of how Aristotle did this.

**The Curved Lines.** While it is clear that the three-term arrays serve as displays of placeholder letters in the *Prior Analytics*, some have wondered whether they should be shown with lines connecting the terms. The Kneales suggested that Aristotle may have used curved lines, above (representing connections in the premisses) and below (representing a connection in the conclusion), to connect the letters.<sup>21</sup>

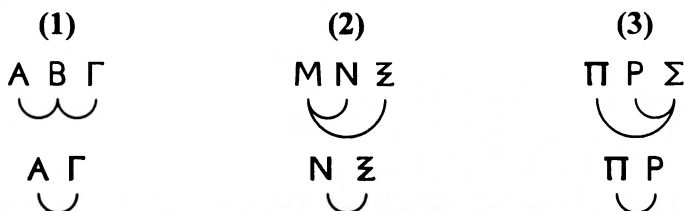
<sup>19</sup> Rose, p. 23.

<sup>20</sup> Rose, p. 21. The Kneales call the pairs of Greek capitals "the skeleton of a general statement," p.68.

<sup>21</sup> William Kneale and Martha Kneale, *The Development of Logic*. Oxford: Oxford University Press, 1962, pp. 71-72. The Kneales cite Ammonius, p. x, but the scholium that Wallies cites in his *praefatio* contains lunes, not curved lines in isolation. There are arcs in some of the Scholia Platonica; cf. Robert S. Brumbaugh, "Logical and mathematical symbolism in the platonic scholia," *Journal of the Warburg and Courtauld Institutes*, XXIV: 45-58, 1961; see especially Plates a-IIIj, bIII, and Vh. J. A. Mulhern, whose indispensable help in the preparation of this paper's graphics I wish to acknowledge, has suggested to me that Aristotle may merely have made a gesture, which his students recorded as a curved line.

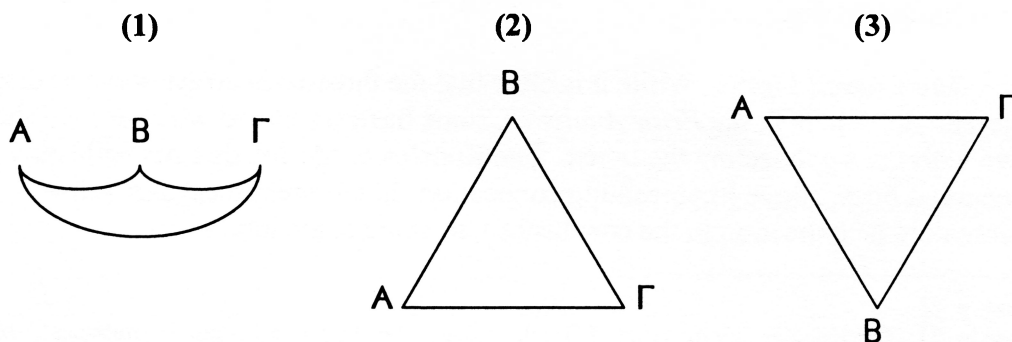


Rose suggested that these lines would have been all below and the conclusion displayed separately from the premisses:



Here arcs under the letters are used to show how the terms are connected in the premisses (first line) and in the conclusion (second line). While this suggestion is plausible, evidence for it in Aristotle and the early commentators, such as Alexander of Aphrodisias, is slim, since these texts contain neither arcs like these nor the lunes and triangles appearing later, and since Alexander does not seem aware of these diagrams.<sup>22</sup>

**The Lune and Triangles.** In the fifth century, however, diagrams illustrating the three figures do appear in scholia to Ammonius' commentary on *Prior Analytics* 1 and are used extensively by his student John Philoponus. These diagrams consist of a lune for the first figure, a normal equilateral triangle for the second, and an inverted equilateral triangle for the third.<sup>23</sup> After Philoponus, the lune and triangles became quite common in mediaeval manuscripts of all sorts, including the Platonic Scholia.<sup>24</sup> To simplify his presentation, Rose replaced the three sets of placeholder letters in the *Prior Analytics* by ABΓ for all three figures. His diagrams idealize the varied extant specimens, preserving the position of the middle term as it is in the three-term arrays, in the middle in the first figure (B), to the left in the second (A), and to the right in the third figure (Γ).



<sup>22</sup> "Alexander seems completely unaware of any such diagrams," Rose, p. 134. Rose cites Greek commentators on Aristotle's *Rhetoric*, who do use arcs under their letter arrays to represent rhetorical syllogisms in the third figure, to support his position.

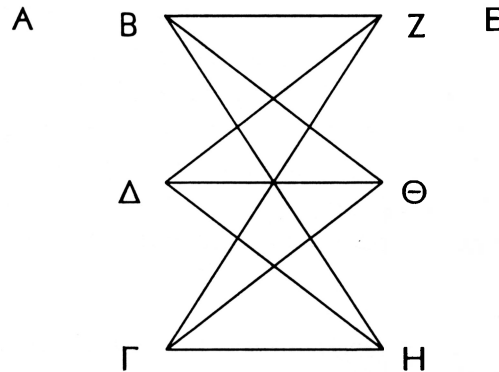
<sup>23</sup> *Ammonii in Aristotelis Analyticorum Priorum Librum I Commentarium*. In *CAG*, edited by Maximilian Wallies. Berlin: G. Reimer, 1899, vol. IV, part VI, especially viii, x-xii, and 39-42, and *Ioannis Philoponi in Aristotelis Analytica Priora Commentaria*, in *CAG*, 1905, vol. XIII, part II, in Rose, p. 133.

<sup>24</sup> Brumbaugh calls them "the standard arc-and-triangle syllogism figures," p. 46.

**The Proportion Diagrams.** Rose mentions a passage “where Philoponus seems to be defending the naturalness of such diagrams.”<sup>25</sup> In this passage, Philoponus is undoubtedly discussing arrangement (τάξις) of the three figures but his arrangements vary from the standard lune and triangle diagrams because he concludes: “Therefore according to the first figure the middle term is drawn in a straight line, according to the second it is drawn above, according to the third it is drawn below.” Here the first figure is “straight,” not a lune; the second has the middle term above, not below; and the third has the middle term below, not above. It seems more likely that Philoponus has in mind proportion diagrams employing lines, whose logical use is discussed by Einarson.<sup>26</sup> In these diagrams, also displayed by Ross, *APPA*, ad 25b26ff, to which Philoponus’ commentary is directed, the length of the lines answers to the generality of the terms, as they appear next to the vertically arranged placeholder letters. In the first figure, the straight line for the middle term is in the middle; in the second, it is above; and in the third, it is below, all as Philoponus’ passage has it.

First Figure	Second Figure	Third Figure
A <u>major</u>	(A) <u>M middle</u>	(A) <u>Π major</u>
B <u>middle</u>	(B) <u>N major</u>	(B) <u>Ρ minor</u>
Γ <u>minor</u>	(Γ) <u>Ξ minor</u>	(Γ) <u>Σ middle</u>

**The Pons Asinorum.** Alexander describes, under the rubric of diagram (διάγραμμα), what became known in the Middle Ages as the *pons asinorum*.<sup>27</sup> The diagram itself is not preserved in the text, but it appears, with Alexander’s examples, in Philoponus’ commentary and is found not only in our oldest Greek manuscript of the *Prior Analytics* but also in a scholium to an early Latin translation, possibly by Boethius, and in at least a hundred Latin MSS. of that work.<sup>28</sup>



<sup>25</sup> Philoponus, 65:14-23, noted in Rose, p. 133, note 2.

<sup>26</sup> Benedict Einarson, “On Certain Mathematical Terms in Aristotle’s Logic: Part II,” *The American Journal of Philology*, Vol. 57, No. 2 (1936), pp. 151-172; see pp. 166-169.

<sup>27</sup> Alexander, *In An. Pr.*, CAG, ii (i), p.301:

σαφηνείας δὲ χάριν τῶν λεγομένων ὑπογράφωμεν τὰς τε ἐκλογὰς καὶ τὸ διάγραμμα ὅλον καὶ τοὺς συλλογισμούς, πῶς τε καὶ ἐκ τίνων γίνονται.

<sup>28</sup> Kneale and Kneale, p. 186; cf. L. Minio-Paluello, “A Latin Commentary (? translated by Boethius) on the *Prior Analytics*, and its Greek Sources,” *Journal of Hellenic Studies*, lxxvii, part I (1957), pp. 93-102. Philoponus, p. 274.

The *pons asinorum* illustrates Aristotle's own method of finding the middle term, *inventio medii* to the Latin Middle Ages, with a practical aim, as he observes:

We must now state how we may ourselves always have a supply of syllogisms in reference to the problem proposed and by what road we may reach the principles relative to the problem: for perhaps we ought not only to investigate the construction of syllogisms, but also to have the power of making them.<sup>29</sup>

In *Prior Analytics* i.27-28, Aristotle details a technique, in his view the only one required, for finding syllogisms to prove conclusions of the four different kinds by finding a middle term. Then ensues what Smith considers a proof of the foregoing,<sup>30</sup> in which Aristotle assigned placeholder letters--A, B, Γ, Δ, E, Z, H, Θ--to the various groups of characters. Aristotle has a theoretical aim here as well, as Smith observes, in that his method in the *Prior Analytics* 1.27-28 "in the terminology of modern logic . . . is comparable to a *decision procedure* for deductive systems."<sup>31</sup> Aristotle's method provides for any predicate whatsoever, any subject whatsoever, and any well-formed formula of his system combining those terms with anything that follows from them, that they follow from, or that is extraneous to them. It thus exhausts all the possible premisses of his syllogistic, and is viewed by him as conclusive.

In the thirteenth century a mnemonic verse had been composed on the placeholder letters, the first eight in the Latin alphabet replacing the first eight in the Greek, to distinguish good connections from bad in the diagram.

FaCia CoGenti DeFert HeBere GraDendo GalBa valent,  
sed non constant HeDes FaBar HirCe.<sup>32</sup>

In this verse, as in the more familiar Barbara Celarent, the capital consonants—two per mood-- identify the syllogistic moods by the placeholder letters on the diagram of the *pons asinorum*. The intervening vowels give the quantity and quality of the propositions, according to the mediaeval A, E, I, O designations. The figure of each syllogism is indicated on the diagram in Philoponus' version. The verses, which treat of nine moods—six making syllogisms and three failing to do so, make it clear that their

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<sup>29</sup> *An. Pr.* 43a20-24, quoted in I. M. Bochenski, *A History of Formal Logic*, trans. Ivo Thomas (New York: Chelsea Publishing Company), 1970, p. 81.

<sup>30</sup> Smith, p. 152, note to 43b39-44a11, but referring to 44a11-35.

<sup>31</sup> Smith, p. 150, *ad* 43b1-11. It should be noted here that Aristotle does not have in mind a contemporary definition of a decision procedure as a mechanical method for conclusively testing any member of an infinite set for having or lacking a certain property.<sup>31</sup> The notion of infinite sets is not unknown to Aristotle but is foreign to him, since he is interested in demonstration and takes the view that infinite series cannot be understood.

<sup>32</sup> This verse is in Minio-Paluello, p. 98, n. 7. Minio-Paluello gives "FaBer" for one of the asyllogistic moods, but this is impossible, since *An. Pr.* 44b31-32 and Philoponus agree that both premisses are affirmative and Philoponus has both universal. Bochenski gives some different mood names, following George of Brussels, and indicating different quantities but the same order of placeholder letters, pp. 220-221.

inventors had in mind all nine lines of Philoponus' diagram. The correspondence of the verses to the diagram will be shown below.

Did conditions in the gymnasia and teaching practice in Aristotle's time make it likely that he used diagrams when he taught syllogistic? On what surfaces did he write? Was Aristotle influenced by what mathematicians did? Henry Jackson thought it very likely that Aristotle used diagrams, as he remarked:

... we may safely conjecture that there was, not a black board, but a white one, a λεύκωμα [a tablet covered with gypsum, used as a notice board, etc. *LSJ*]: for sometimes, e.g. *NE* v 5 § 8 and § 12 and *prior analytics* 52a16, we have the description of a diagram, and in some instances the MSS. reproduce it. Indeed I think that such diagrams ought to be faithfully reproduced in the texts as a tradition dating from Aristotle himself.<sup>33</sup>

Hugh Tredennick, the Loeb translator of the *Prior Analytics*, was of the same opinion:

It is worth noting. . . that the use of the words ὅρος (bound or limit), ἄκρον (extreme) and μέσον (middle) to describe the terms, and of διάστημα (interval) as an alternative to πρότασις or premiss, suggests that Aristotle was accustomed to employ some form of blackboard diagram, as it were, for the purpose of illustration.<sup>34</sup>

Something like a black- or white-board may have been used, but the Greeks also employed many other media to record things. Papyrus and vellum were available but were very expensive. Pupils wrote on erasable wax tablets, and on bits of pottery and pieces of stone, because these latter were readily available and were cheap or free.<sup>35</sup> Numbers of well-developed architectural drawings on stone have been discovered, dating from the 6<sup>th</sup> century BC forward, by Haselberger, who has stated that it would have been odd if Aristotle and his school had not made use of diagrams in their teaching and research because the practice was so advanced and frequent among architects and craftsmen.<sup>36</sup> Einarson has in mind lines on paper or in sand when he writes "The contemporary diagrams with which the doctrine of harmonics and proportion was illustrated were probably horizontal lines, of equal, or what is more likely, varying length, lying directly above or below each other, representing the ὅποι, while the

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<sup>33</sup>Henry Jackson, "Aristotle's Lecture-Room and Lectures," *The Journal of Philology* 35 (1920), 191-200, p.193. On the whiteboard, cf. H. I. Marrou, *A History of Education in Antiquity*, trans. George Lamb (New York: The New American Library), Additional Note 17, p. 519.

<sup>34</sup>*Prior Analytics*, translated by Hugh Tredennick, Loeb Classical Library (Cambridge, MA: Harvard University Press), 1962, p. 184

<sup>35</sup>K. Dale Foster, *The Classroom in Ancient Times* (Unpublished dissertation, Halifax, Nova Scotia: Dalhousie University), 1994.

<sup>36</sup>Lothar Haselberger, pers. comm.

distances between them may originally have been thought of as representing the διαστήματα." <sup>37</sup> Mathematical diagrams even appear on coins. <sup>38</sup>

Rose was not convinced that Aristotle had used anything beyond the three-term array, declaring: "There is no textual evidence that Aristotle used curved lines, triangles, or any other special symbolism." <sup>39</sup> He does admit the probability of some use of lines by the ancient authors:

The word διάστημα, which Aristotle occasionally uses for "statement," means a line or distance or interval, and thus seems to be a natural way of referring to the statement that connects two terms. It seems probable that both Plato and Aristotle connected terms with lines. <sup>40</sup>

Rose also cites Sir William Hamilton on the likelihood of lines and Aristotle's use of geometrical terminology:

A proposition (διάστημα, intervallum, πρότασις, literally protensio, the stretching out of a line from point to point, is a mutual relation of two terms (ὅροι) or extremes (ἄκρα). This is therefore well represented,—the two Terms, by two letters, and their Relation, by a line extended between them. <sup>41</sup> He also frequently—indeed as often as he can, borrows his Logical nomenclature from the language of Geometry; as ὅρος, ἄκρον,—διάστημα, πρότασις,—σχῆμα. Even the word Syllogism (συλλογισμός) is mathematical—a computation. <sup>42</sup>

Ross, also, is sensitive to Aristotle's terminology having, as he puts it, "a mathematical air," and he reasons from this that "It is not unlikely that he represented each figure of the syllogism by a different geometrical figure, in which the lines stood for propositions and the points for terms." <sup>43</sup>

The question remains: if these diagrams were used in Aristotle's school, what happened to them, so that they do not reappear until almost a millennium later? Almost all ancient works have a checkered history of transmission, but the notorious vicissitudes of Aristotle's works must receive extraordinary consideration. The *Prior Analytics*, along with most of the rest of the extant Aristotelian material, consists of work not published externally by Aristotle but rather of lecture notes—his own or his

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<sup>37</sup> Einarson, p. 166.

<sup>38</sup> Benno Artmann, "Euclid's *Elements* and its Prehistory," in *ΠΕΡΙ ΤΩΝ ΜΑΘΗΜΑΤΩΝ*, ed. Ian Mueller, in *Apeiron*, XXIV, 4 (December 1991), pp. 1-47.

<sup>39</sup> Rose, p. 23.

<sup>40</sup> Rose, p. 10. Cf. "A premiss was probably represented by a line joining the letters chosen to stand for the terms." Tredennick, p. 184.

<sup>41</sup> Hamilton, Sir William. *Discussions on Philosophy and Literature, Education and University Reform*, 3<sup>rd</sup> edition (Edinburgh and London: William Blackwood and Sons), 1866, p. 654, quoted in Rose, p. 10, n. 25.

<sup>42</sup> Hamilton, p. 663, quoted in Rose, p. 10, n. 25.

<sup>43</sup> Sir David Ross, *Aristotle*, 5<sup>th</sup> edition (London: Methuen & Co, Ltd), 1949, p. 33.

students'—internal to his teaching and research enterprises. On Aristotle's death in 322 BC, his manuscripts passed to Theophrastus, who kept them for more than 34 years in Athens. Theophrastus willed his own library with Aristotle's to his pupil Neleus, who removed the books to his home at Scepsis in the Troad. Here Neleus and his family were obliged to conceal the volumes from the Attalid dynasty, who would have seized them for their library at Pergamon. About 100 BC, Apellicon of Teos bought them from the Neleids and brought them back to Athens. After Sulla's capture of Athens in 86 BC, they were brought to Rome, where they were studied by Tyrannion and Andronicus of Rhodes, as Strabo reports.<sup>44</sup> Unfortunately, Neleus' heirs had hid the collection in a pit, where they had been much damaged. Again according to Strabo,<sup>45</sup> Apellicon, a bibliophile, and Tyrannion, a grammarian, made and caused to be made unskillful corrections and supplements. Andronicus of Rhodes did a somewhat better job, and it is on his edition that the modern *corpus Aristotelicum* rests, although the earliest manuscripts are much later, dating from the 9<sup>th</sup> and 10<sup>th</sup> centuries AD.

This history shows us, first, that, if there were diagrams originally, there was not a standard published text to put them in. As we all know, with course notes in a library, different--more or less complete-- versions often exist side by side in different formats, especially from year to year. Next, assuming that, from at least near the beginning, there were diagrams to illustrate the syllogistic, there was ample opportunity for them to have been lost.

***From the Three-term Arrays to the Lune and Triangles.*** The three-term array, established convincingly by Rose, shows clearly the relation of the terms and the difference of the three figures one from another. Rose has also shown that, if there were lines, they were most likely downward-curving lines, not a combination of upward and downward lines as suggested by the Kneales, but he argues that there is no textual evidence that Aristotle used these lines or other devices.<sup>46</sup> Here, nevertheless, Rose does have textual evidence of a kind, from Aristotle's use of the expression ὑπάρχειν, one of the spatial expressions prominent in Aristotle's development of his syllogistic. The expression, often translated 'belongs to' or 'applies to,' is compounded of the preposition ὑπό, 'under,' and the verb ἄρχειν, 'to begin, be the leader of.' It might mean that B leads under A, and Γ leads under B, so that Γ leads under A, and so on. It thus contains a spatial element, as we have come to expect in Aristotle's logical vocabulary, and that spatial element is 'under' and thus supports the arcs under the place-holding letters. The figure or σχῆμα, basis of Aristotelian syllogistic, is designated by the same word as geometric figure, suggesting that in his research leading up to the writing of the *Prior Analytics* Aristotle represented the syllogistic figures by geometric figures.

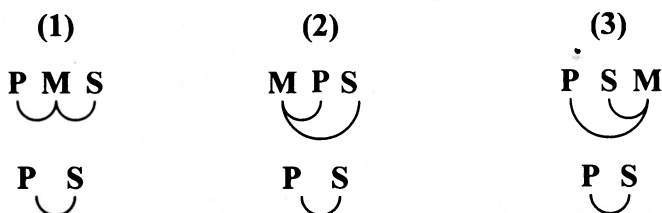
<sup>44</sup> John Edwin Sandys, *A History of Classical Scholarship*, 3 vols. 3<sup>rd</sup> edition (New York and London: Hafner Publishing Company), 1967, pp.85-86.

<sup>45</sup> XIII, 1.54, quoted in John Patrick Lynch, *Aristotle's School* (Berkeley: University of California Press), 1972, p.201.

<sup>46</sup> The Kneales, agreeing with Rose's position on Aristotle's use of three-term linear arrays for all three figures, argue that the diagrams make it obvious why Aristotle assumed that there could be only three syllogistic figures.

Aristotle uses σχήμα, along with μορφή, ‘shape,’ in the *Categories* in his discussion of figure as one of the kinds of quality.<sup>47</sup> Here his examples are qualities of geometric figures—triangular and quadrangular. The three schemata of the *Prior Analytics* are schemata in the modern sense, syntactic strings with blanks as placeholders.<sup>48</sup> The figures are schemata, from which are generated the moods, also schemata, which are argument-text templates, from which in turn are generated the indefinitely many syllogisms, which makes the syllogisms instances of schemata. The Aristotelian figures are a limiting case of schema, as being all blanks, with no normal characters between the blanks in the string. The placeholder characters, nevertheless, are in a spatially-ordered arrangement with regard to one another.

The Aristotelian syllogism produces its conclusion by having a middle term common to its two premisses. In Aristotle's three term arrays, the middle term is in the middle (between the two extremes) in the first or perfect figure, which is called 'first' or 'perfect' partly for this reason. Since there are only three placeholders in each figure, there are only two other possible positions for the middle term in the other two figures, to the left and to the right. This gives us the three arrangements that we have already seen, but now with P standing for 'predicate [of the conclusion],' S for 'subject [of the conclusion],' and M for 'middle':

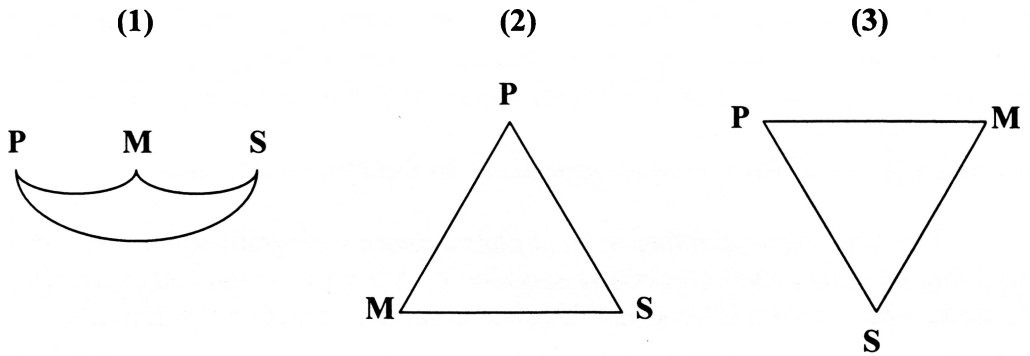


It is not difficult to see how the three-term arrays for the three figures evolve into a lune and two triangles, although this evolution springs from the single-line three-term arrays, not requiring the conclusion to be written as a separate line. The first figure is obvious, as it merely requires joining P and S of the conclusion with a longer curved line that joins the shorter arcs at the ends of the three-term array, giving a three-sided figure, although all the sides are curved. For the second figure: join the conclusion terms with a line in the three-term array, thus giving a three-sided figure, straighten out all the lines, and drop the leftmost and rightmost terms, the middle term remaining on the left but now on the bottom. For the third figure, proceed as for the second figure, and drop only the center term, leaving the middle term still at the right and still at the top. The diagrams are at least mnemonic in purpose, so they need to be distinct from one another. The “standard” three diagrams accomplish this in the simplest way, with three three-sided figures, preserving the position of the middle term in the figures as middle, left, and right.

<sup>47</sup> 10a; cf. Thomas Heath, *Mathematics in Aristotle* (Oxford at the Clarendon Press), 1949, p. 19.

<sup>48</sup> John Corcoran, "Schema", *The Stanford Encyclopedia of Philosophy* (Summer 2004 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/sum2004/entries/schema/>.





Thus it is plain how the diagrams may have developed from the schemata and how they are used to distinguish and recall some features of the limiting case schemata that are the three-term arrays of placeholder letters. Their next use is to provide argument-templates for the moods, since the three-term arrays leave no room for quantity, quality, or modality. These last, on the other hand, are easily written along the lines of the diagrams, and this is, in fact, the way that they appear in the manuscripts.<sup>49</sup> Sometimes the terms are given as well, which yields an instance of the mood schema, that is to say, a syllogism proper. Thus we see that:

- the Greek capitals give us placeholders for the terms
- the three-term array of Greek capitals gives us a limiting-case schema for the three figures of syllogism, with the positions of the placeholders indicated
- the three-term array with the linking curved lines specifies which placeholders are related to one another
- the lune and triangles show us how the placeholders or terms are related, make it easier to remember how the three figures are distinguished, and provide handy spaces for specifying the quantity, quality, and modality of the relations

### From the Text to the Proportion Diagrams

Einarson urges that it is probable that Aristotle used the proportion diagrams because of their common use of letters of the alphabet for the terms as extremes of the lines.<sup>50</sup> He adduces *Nichomachean Ethics* 1131b1-3 (a little before Jackson's example at 1133a), dealing with the proportion of desert and reward, as evidence of Aristotle's discussion of a diagram in which terms were represented as lines, "as the feminine article shows [since the Greek word for 'line,' γραμμή, is feminine, and such expressions as ἡ AB, naming lines by their extremities, are widely used]".<sup>51</sup> That these were horizontal, parallel lines and not segments of a continuous line is indicated by the placing of line B twice, by the fact that a man and his reward are not two quantities of the same kind, and

<sup>49</sup> This is true both of Philoponus, see Wallies' preface, p. xxxvii, and of the Scholia Platonica, see Brumbaugh, p. 51, and Figures IIIs, IIIr, and Ve.

<sup>50</sup> Einarson, p. 166, n. 64. He refers to Ptolemy, *Harmonica*, I, 5; Euclid, *Elementa*, V and VII-IX, *passim*; and the *Sectio Canonis*.

<sup>51</sup> Einarson, p. 167; cf. p. 166, n. 68 and the *Sectio Canonis* and Euclid, *Elements*, V and VII-X.

by Aristotle's frequent use of ἄνω, 'above,' and κάτω and ὑπό, 'below,' of the scale of predication. Einarson suggests that the use of such a diagram can be confirmed from the discussion of conversion in all three figures in *Prior Analytics* II, 8-10.<sup>52</sup>

### From the Proof of the Decision Procedure to the *Pons Asinorum*

We have seen the rather ancient *pons asinorum* diagram and one set of the surviving late mediaeval verses that expound it. We will use the verses, which embody Aristotle's placeholder letters, and his own remarks to generate the diagram.

Announcing in 44a13 that perhaps the preceding, in which he has detailed his decision procedure for syllogistic, will be clearer, Aristotle introduces eight placeholder letters in his proof of the foregoing decision procedure. Would any teacher ask his students to remember eight placeholder letters without writing them down? A stands for any predicate you like, within the formation rules of the *Prior Analytics*, and is written on the left, where Aristotle always writes his predicates because of the oblique constructions with ὑπάρχειν. E stands for any subject that you like, within the same strictures, and is written on the right, where Aristotle always writes his subjects. B stands for all that follows from A and is written next to it, at the top. Γ stands for all that from which A follows and is written next to A, but at the bottom, as the contrary of B at the top. Δ stands for all that cannot apply to A, and it goes on the A side, but in the middle, as having no answering group for the A's. Likewise for the subject E: on the right side, Z at the top stands for what follows from E, H at the bottom for that from which E follows, and Θ in the middle for what cannot apply to E.

The placeholder letters thus arrayed, the lines are filled in by Aristotle's examples, and the diagram fairly draws itself. Aristotle's cases in which syllogisms can be effected are as follows (44a18-25):

1. Z-Γ (FaCia in Minio-Paluello's mnemonic), a first figure syllogism.
2. Γ-H (CoGenti), third figure.
3. Δ-Z (DeFert), first and second figures.
4. Θ-B (HeBere), third figure.
5. H-Δ (GraDendo), first and third figures [Philoponus/Bochenski]
6. H-B (GalBa), first and third figures [Philoponus/Bochenski]

He goes on (44b25-38) to review the remaining three cases, which do not yield syllogisms. These are:

7. Z-B (FaBar), second figure, because of trying to conclude from two universal affirmative premisses.

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<sup>52</sup> Einarson, pp. 167-168.

8.  $\Theta\text{-}\Gamma$  (HirCe), first figure, because of trying to conclude with a negative minor premiss.
9.  $\Theta\text{-}\Delta$  (HeDes), first or second figures, because of trying to conclude from two negative premisses.

Aristotle's proof thus has accounted exhaustively for all components of premisses by The placeholder letters, for all premisses by the connection of the terms using the lines, and for all four kinds of conclusions.

Bochenski has suggested that, if diagrams are used in logical writing, their purpose is expository, probative, or mnemonic, or they may belong to methodology and be outside logic but relevant to it.<sup>53</sup> The lune and triangle diagrams are at least mnemonic, as they help to keep the figures distinct. They are also expository, as they display the relations between the terms, while the linear arrays emphasize the terms related. They also provide a handy and efficient format for indicating quantity, quality, or modality—features of syllogisms suppressed in the very concise three-term arrays.<sup>54</sup> It appears that the three standard syllogistic diagrams are probative as well, because they are proof forms, in Corcoran's sense,<sup>55</sup> that is to say, that they are gapless and rigorous, containing the maximum amount of logical detail. Aristotle's perfect syllogisms are gapless, and his reductions to the first figure are likewise gapless, and the lune and triangle diagrams provide for the display of their maximum amount of logical detail.<sup>56</sup>

Bochenski thought that the *pons asinorum* was extra-logical and merely methodological-- a help in finding the middle term. As methodological, this diagram is, of course, mnemonic as well, especially when reinforced by the mnemonic verses. If, however, as Smith has suggested, the *inventio medii* is a decision procedure, and the *pons asinorum* is an illustration of its proof, then the *pons asinorum* is a proof-form as well and thus not outside Aristotle's logic. Interestingly, the *pons asinorum* codifies not only the logically rigorous proofs that make syllogisms, but it also codifies the three remaining possibilities that do not make syllogisms.

### Subject-Connector-Predicate

It is clear that the premisses and conclusions of traditional syllogistic are subject-connector (copula)-predicate affairs. Some have expressed the view that the premisses of the *Prior Analytics* are likewise to be understood as made up of a subject term, a predicate term, and one of the four connectors, giving universal affirmative, universal negative, particular affirmative, and particular negative.<sup>57</sup> It remains to be seen, however, whether this conclusion is correct. From the foregoing, two reasons to suspect that it is incorrect spring to mind: first, that the syntax of the *Categories*, *On Interpretation*, and

<sup>53</sup> Bochenski, pp. 143-144.

<sup>54</sup> Kneales, p. 187, "triangular figures . . . to explain syllogisms and their reduction *per impossibile*."

<sup>55</sup> John Corcoran, "Three Logical Theories," *Philosophy of Science*, Volume 36, Number 2, June 1969, pp. 153-177; see especially pp. 162-171.

<sup>56</sup> Cf. Peirce, *CP* 2.601, 2.782; cf. 3.363.

<sup>57</sup> Corcoran 2003, 268.

*Topics* presents a function-argument analysis of propositions, and, second, that since Aristotle's constructions with it are oblique, it is not entirely clear that ὑπάρχειν is simply a connector, in the equational or copulative sense. To make this perfectly clear, let us parse a passage in which some of Aristotle's formalized propositions appear, along with concrete substitutions for the variables (*An. Pr.* 25a22-26; the clauses displayed separately below are continuous, in the order presented, in Aristotle's text):

εἰ δέ γε τὸ Α τινὶ τῶν Β μὴ ὑπάρχει,  
but if A does not apply to some one of the Bs,

Note here that Greek definite articles (in the neuter gender) are prefixed to the letter variables to indicate their grammatical role and, perhaps, to indicate that they are being mentioned, not used. Thus, τὸ Α indicates that Α is nominative and singular, while τῶν Β indicates that Β is genitive and plural, hence "of the Bs." The demonstrative pronoun τινὶ is singular and dative, hence "to some one."

οὐκ ἀνάγκη καὶ τὸ Β τινὶ τῷ Α μὴ ὑπάρχειν  
it is not necessary that Β not apply to some Α

Here, Β is singular and nominative while Α is singular and dative.

οἷον τὸ μὲν Β ἐστὶ ζῷον τὸ δὲ Α ἄνθρωπος·  
for example, Β is 'animal,' Α 'man'

Here both Α and Β are nominative and singular and 'is' (ἐστὶ)—not any inflection of ὑπάρχειν—is used for the substitution of concrete terms for the variables. The concrete terms are themselves nominative and singular.

ἄνθρωπος μὲν γὰρ οὐ παντὶ ζῷῳ [*scil.* ὑπάρχει],  
for 'man' does not apply to every 'animal,'

Here, 'man' is again in the nominative singular, while 'animal' is in the dative singular. Tredennick, the Loeb translator, puts 'man' in single quotes, but not 'animal.' It is clear that ἄνθρωπος ('man') is being mentioned here, and since there is nothing in the text to indicate that ζῷῳ ('animal'), which Aristotle has substituted for a variable in exactly the same way, should be treated differently, I have enclosed it in single quotes as well, to indicate that it is being mentioned rather than used and to emphasize that ὑπάρχειν operates on the terms, not on the designata of the terms.

ζῷον δὲ παντὶ ἀνθρώπῳ ὑπάρχει.  
but 'animal' applies to every 'man.'

Here, 'animal' is nominative and singular, while 'man' is dative singular, and I argue, by the same reasoning as above, that both are terms being mentioned.

In Greek as in English, expressions on either side of ‘is’ or other parts of the verb ‘to be’ typically are in the same case. Ancient Greek, an inflected language in which spelling tells what role a word has in a sentence, has peculiarities of word order that are not reflected in modern English, an analytic language in which word order, not spelling, tells what role a word has in a sentence. As has been argued earlier, Aristotle needed a construction which shows clearly which term is the predicate and which is the subject. Taken together with his stricture that the predicate must always be of a higher generality than the subject, his choice of ὑπάρχειν with the dative not only makes it unlikely that premisses in the *Prior Analytics* contain a copula but also may explain why there are no identity statements among those premisses.<sup>58</sup>

Again, Aristotle’s choice of ὑπάρχειν shows, as Patzig remarked, that he shunned an equational interpretation of his propositions. Had Aristotle wanted to suggest a sort of default ‘S is P’ interpretation of the propositions of his syllogistic, the Greek verb ‘to be’ was ready to hand and entirely natural. There was no need to introduce a fancy substitute if this was all that he meant. Aristotle does introduce technical terms or technical uses of otherwise ordinary terms, but always to some serious purpose. Perhaps the grammarians are at the root of the copulative interpretation of Aristotle. Ὑπάρχειν comes into Latin, especially late Latin, as *substare*, ‘to stand under’ and we eventually get (in 1559, according to the *Shorter OED*) ‘substantive verb’ for late Latin *verbum substantivum*, which in turn translates Greek ῥῆμα ὑπαρκτικόν. For Priscian (p. 812, *fin.*), the substantive verb was the verb ‘to be,’ but one is left to ponder what is the ‘substantive’ sense or use of ‘to be.’ (Cf. Apollonius Dyscolus, *de Syntaxi* 65:13 and *De Pronominibus* 25:2.)

### The Intended Interpretation of Aristotle’s Syllogistic

Cohen and Nagel say that Aristotle’s syllogistic *may* be interpreted as a calculus of classes, which undoubtedly is true, since it has been so interpreted.<sup>59</sup> It remains to be seen, however, whether there is good evidence that this was Aristotle’s intended interpretation.

To begin, it may be observed that the modern notion of ‘class’ is indeed modern—entering English usage in 1664.<sup>60</sup> The English word ‘class’ derives from Latin *classis*, whose first meaning is a census class (or tax bracket, under Servius Tullius, sixth king of Rome, traditionally 578-535 B.C.); the Latin in turn derives from Greek κλησις, from καλέω, ‘call’ or ‘summon.’ Κλησις does not occur in the *corpus Aristotelicum*.<sup>61</sup> Indeed, the modern notion of class as a collection of objects is somewhat foreign not only to Aristotle’s thought but to ancient Greek thought generally. Aristotle does indeed give

<sup>58</sup> Note to Einarson

<sup>59</sup> M. Cohen and E. Nagel, *Introduction to Logic*, 2<sup>nd</sup> ed. (Indianapolis: Hackett Publishing Company), 1993, p. 121. Cf., e.g., Corcoran 2003, 269, “. . . Aristotle’s choice of a class of propositions . . . whose connectors include the simplest logical relations among species, namely inclusion and exclusion.”

<sup>60</sup> *Shorter Oxford English Dictionary*, *ad loc.*

<sup>61</sup> H. Bonitz, *Index Aristotelicus*.

us words for ‘kind’—εἶδος and γένος—but these are part of an apparatus of definition, not a calculus of classes. For him and his contemporaries, other words that might be rendered ‘class’ in English, like τάξις, refer first to the *arrangement* of soldiers, say, and then only later and derivatively to the collection of soldiers thus arranged. In his *Politics*, Aristotle does use τέλος, an expression that has been rendered ‘class,’ when he reports that Solon appointed all the offices from the Five-hundred bushel class, the Teamsters, and the Knighthood, while the Thetes or day labourers were excluded from office (*Pol.* 1274a20-22). The Peripatetic *Constitution of the Athenians* (vii) reports that Solon divided the citizens of Athens by worth into the four τέλη discussed in the *Politics*.<sup>62</sup>

Rose, in discussing the method of division exhibited in Plato’s *Statesman*, declares

Since it is convenient to treat the original idea which is first divided as a more comprehensive class or genus, and the ideas which fall under it as species or subclasses, there is a danger that we will misconstrue the nature of Plato’s ideas. They are not classes in the modern sense. . . . Classes . . . are the barest of abstractions . . . The modern notion of class or set is not to be read into Plato’s writings. (We shall see that it is not to be read into Aristotle’s writings either.)”<sup>63</sup>

Rose, after having pointed out that there is no reason in most of the arguments considered in the *Prior Analytics* to assume that the major term will be larger than the minor term, goes on to say what he means about not reading a modern notion of class into Aristotle’s writings:

When you get into other figures [than the first], or when you deal with statements that are or may be false, there is no longer any guarantee that the terms or classes will stand in the relation of larger, middle-sized, and smaller. . . . The question of the extension or size of the classes designated by various terms becomes *completely* irrelevant when Aristotle introduces variables for terms. When you investigate syllogistic logic in terms of ABC rather than plants-trees-maples, questions of factual truth and falsity and factual extension, inclusion, etc., become irrelevant.<sup>64</sup>

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<sup>62</sup> J. J. Mulhern, “ΤΟΝ ΑΠΛΩΣ ΠΟΛΙΤΗΝ (Aristotle, *Pol.* 1275a19),” International Conference on Ancient and Medieval Philosophy, Fordham University Lincoln Center, 1<sup>st</sup> November 2003, suggests that Aristotle, for whom τέλος in many cases means ‘end’, in the sense of that at which an agent aims, uses the term here to emphasize that Solon’s division is based on what the various divisions of the population produce in total. He observes further: “Aristotle is not concerned here with classes in either the Boolean or the Marxian sense.”

<sup>63</sup> Rose 1968, 6.

<sup>64</sup> Rose 1968, 11.

Thus it would seem that the burden of proof is on those who would urge that a class interpretation is the primary one intended for Aristotle's syllogistic, not the other way round.

## Conclusion

A formal language was invented by Aristotle and used by him in his lectures. This formal language consisted of Greek capital letters used as placeholders, arrayed in the schemata of the three figures recognized as authentically Aristotle's. In these arrays, arcs under the placeholder letters indicate how the terms are linked in the premisses and conclusion and are read as some inflection of ὑπάρχειν, used by Aristotle as a second-order expression to convey the relation that the terms—not the designata of the terms—of a syllogism have to one another. It is further possible that Aristotle elaborated the three-term arrays with arcs into lune and triangle diagrams like those appearing in Ammonius, Philoponus and the Scholia Platonica. The lune and triangle diagrams are developed easily out of the three-term arrays with arcs, and they are consistent with the figures' being the schemata of their instances the moods, which moods are in turn schemata of the syllogisms themselves. Conditions in Aristotle's classroom make it plausible that the diagrams were in use there, whether by master or by pupil. These diagrams serve as proof forms, as does the *pons asinorum* diagram, obviously constructible from Aristotle's remarks in *Prior Analytics* 1: 27-28. As proof-forms of the syllogistic, the diagrams are not mere addenda to Aristotle's system, which makes it more likely that he developed them himself.

This formal language in the *Prior Analytics* accommodates all the kinds of propositions that are discussed in the *Categories*, *On Interpretation*, and *Topics*. The analysis of Aristotle's formal language presented here corrects the views that his logic uses propositions of the subject-copula-predicate form and that the intended interpretation of his syllogistic is as a logic of class inclusion and exclusion.